Spectral Methods in Fluid Dynamics | A First Course in the Numerical Analysis of Spectral Methods South Asian Edition

Along with finite differences and finite elements, spectral methods are one of the three main methodologies for solving partial differential equations on computers. This book provides a detailed presentation of basic spectral algorithms, as well as a systematical presentation of basic convergence theory and error analysis for spectral methods. Readers of this book will be exposed to a unified framework for designing and analyzing spectral algorithms for a variety of problems, including in particular high-order differential equations and problems in unbounded domains. The book contains a large number of figures which are designed to illustrate various concepts stressed in the book. A set of basic matlab codes has been made available online to help the readers to develop their own spectral codes for their specific applications.

Numerical analysis explains why numerical computations work - or fail. These are mathematical questions, and the book answers in kind, providing students with a very complete and sound presentation of the interface between mathematics and scientific computation. The book does not assume previous knowledge of numerical methods. It includes a large range of exercises, and will be suitable as a textbook at the advanced undergraduate level.

A unified discussion of the formulation and analysis of special methods of mixed initial boundary-value problems. The focus is on the development of a new mathematical theory that explains why and how well spectral methods work. Included are interesting extensions of the classical numerical analysis.

Riemann-Hilbert problems are fundamental objects of study within complex analysis. Many problems in differential equations and integrable systems, probability and random matrix theory, and asymptotic analysis can be solved by reformulation as a Riemann-Hilbert problem. This book, the most comprehensive one to date on the applied and computational theory of Riemann-Hilbert problems, includes an introduction to computational complex analysis, an introduction to the applied theory of Riemann-Hilbert problems from an analytical and numerical perspective, and a discussion of applications to integrable systems, differential equations, and special function theory. It also includes six fundamental examples and five more sophisticated examples of the analytical and numerical Riemann-Hilbert method, each of mathematical or physical significance or both.

Spectral methods are well-suited to solve problems modeled by time-dependent partial differential equations: they are fast, efficient and accurate and widely used by mathematicians and
practitioners. This class-tested 2007 introduction, the first on the subject, is ideal for graduate courses, or self-study. The authors describe the basic theory of spectral methods, allowing the reader to understand the techniques through numerous examples as well as more rigorous developments. They provide a detailed treatment of methods based on Fourier expansions and orthogonal polynomials (including discussions of stability, boundary conditions, filtering, and the extension from the linear to the nonlinear situation). Computational solution techniques for integration in time are dealt with by Runge-Kutta type methods. Several chapters are devoted to material not previously covered in book form, including stability theory for polynomial methods, techniques for problems with discontinuous solutions, round-off errors and the formulation of spectral methods on general grids. These will be especially helpful for practitioners.

A balanced guide to the essential techniques for solving elliptic partial differential equations Numerical Analysis of Partial Differential Equations provides a comprehensive, self-contained treatment of the quantitative methods used to solve elliptic partial differential equations (PDEs), with a focus on the efficiency as well as the error of the presented methods. The author utilizes coverage of theoretical PDEs, along with the numerical solution of linear systems and various examples and exercises, to supply readers with an introduction to the essential concepts in the numerical analysis of PDEs. The book presents the three main discretization methods of elliptic PDEs: finite difference, finite elements, and spectral methods. Each topic has its own devoted chapters and is discussed alongside additional key topics, including: The mathematical theory of elliptic PDEs Numerical linear algebra Time-dependent PDEs Multigrid and domain decomposition PDEs posed on infinite domains The book concludes with a discussion of the methods for nonlinear problems, such as Newton's method, and addresses the importance of hands-on work to facilitate learning. Each chapter concludes with a set of exercises, including theoretical and programming problems, that allows readers to test their understanding of the presented theories and techniques. In addition, the book discusses important nonlinear problems in many fields of science and engineering, providing information as to how they can serve as computing projects across various disciplines. Requiring only a preliminary understanding of analysis, Numerical Analysis of Partial Differential Equations is suitable for courses on numerical PDEs at the upper-undergraduate and graduate levels. The book is also appropriate for students majoring in the mathematical sciences and engineering.

The 1947 paper by John von Neumann and Herman Goldstine, OC Numerical Inverting of Matrices of High OrderOCO (Bulletin of the AMS, Nov. 1947), is considered as the birth certificate of numerical analysis. Since its publication, the evolution of this domain has been enormous. This book is a unique collection of contributions by researchers who have lived through this evolution, testifying about their personal experiences and sketching the evolution of their respective subdomains since the early years. Sample Chapter(s). Chapter 1: Some pioneers of extrapolation methods (323 KB). Contents: Some Pioneers of Extrapolation Methods (C Brezinski); Very Basic Multidimensional Extrapolation Quadrature (J N Lyness); Numerical Methods for Ordinary Differential Equations: Early Days (J C Butcher); Interview with Herbert Bishop Keller (H M Osiniga); A Personal Perspective on the History of the Numerical Analysis of Fredholm Integral Equations of the Second Kind (K Atkinson); Memoires on Building on General Purpose Numerical Algorithms Library (B Ford); Recent Trends in High Performance Computing (J J Dongarra et al.); Nonnegativity Constraints in Numerical Analysis (D-H Chen & R J Plemmons); On Nonlinear Optimization Since 1959 (M J D Powell); The History and Development of Numerical Analysis in Scotland: A Personal Perspective (G Alistair Watson); Remembering Philip Rabinowitz (P J Davis & A S Fraenkel); My Early Experiences with Scientific Computation (P J Davis); Applications of Chebyshev Polynomials: From Theoretical Kinematics to Practical Computations (R Piessens). Readership: Mathematicians in numerical analysis and mathematicians who are interested in the history of mathematics.

Revised and updated, this second edition of Walter Gautschi's successful Numerical Analysis explores computational methods for problems arising in the areas of classical analysis, approximation theory, and ordinary differential equations, among others. Topics included in the book are presented with a view toward stressing basic principles and maintaining simplicity and teachability as far as possible, while subjects requiring a higher level of technicality are referenced in detailed bibliographic notes at the end of each chapter. Readers are thus given the guidance and opportunity to pursue advanced modern topics in more depth. Along with updated references, new bibliographical notes, and enhanced notational clarity, this second edition includes the expansion of an already large collection of exercises and assignments, both the kind that deal with theoretical and practical aspects of the subject and those requiring machine computation and the use of mathematical software. Perhaps most notably, the edition also comes with a complete solutions manual, carefully developed and polished by the author, which will serve as an exceptionally valuable resource for instructors.
This is a very lucid introduction to spectral methods emphasizing the mathematical aspects of the theory rather than the many applications in numerical analysis and the engineering sciences. The first part is a fairly complete introduction to Fourier series while the second emphasizes polynomial expansion methods like Chebyshev's. The author gives rigorous proofs of fundamental results related to one-dimensional advection and diffusion equations. The book addresses students as well as practitioners of numerical analysis.

This volume is designed as an introduction to the concepts of modern numerical analysis as they apply to partial differential equations. The book contains many practical problems and their solutions, but at the same time, strives to expose the pitfalls—such as overstability, consistency requirements, and the danger of extrapolation to nonlinear problems methods used on linear problems. Numerical Methods for Partial Differential Equations, Third Edition reflects the great accomplishments that have taken place in scientific computation in the fifteen years since the Second Edition was published. This new edition is a drastic revision of the previous one, with new material on boundary elements, spectral methods, the methods of lines, and invariant methods. At the same time, the new edition retains the self-contained nature of the older version, and shares the clarity of its exposition and the integrity of its presentation.

Key Features:
- Material on finite elements and finite differences have been merged, and now constitute equal partners
- Additional material has been added on boundary elements, spectral methods, the method of lines, and invariant methods
- References have been updated, and reflect the additional material
- Self-contained nature of the Second Edition has been maintained

Extensively updated new edition including new chapters on emerging subject areas: geometric numerical integration, spectral methods and conjugate gradients.

Spectral Methods Using Multivariate Polynomials on the Unit Ball is a research level text on a numerical method for the solution of partial differential equations. The authors introduce, illustrate with examples, and analyze 'spectral methods' that are based on multivariate polynomial approximations. The method presented is an alternative to finite element and difference methods for regions that are diffeomorphic to the unit disk, in two dimensions, and the unit ball, in three dimensions. The speed of convergence of spectral methods is usually much higher than that of finite element or finite difference methods. Features:
- Introduces the use of multivariate polynomials for the construction and analysis of spectral methods for linear and nonlinear boundary value problems
- Suitable for researchers and students in numerical analysis of PDEs, along with anyone interested in applying this method to a particular physical problem
- One of the few texts to address this area using multivariate orthogonal polynomials, rather than tensor products of univariate polynomials.

This book presents the basic algorithms, the main theoretical results, and some applications of spectral methods. Particular attention is paid to the applications of spectral methods to nonlinear problems arising in fluid dynamics, quantum mechanics, weather prediction, heat conduction and other fields. The book consists of three parts. The first part deals with orthogonal approximations in Sobolev spaces and the stability and convergence of approximations for nonlinear problems, as the mathematical foundation of spectral methods. In the second part, various spectral methods are described, with some applications. It includes Fourier spectral method, Legendre spectral method, Chebyshev spectral method, spectral penalty method, spectral vanishing viscosity method, spectral approximation of isolated solutions, multi-dimensional spectral method, spectral method for high-order equations, spectral-domain decomposition method and spectral multigrid method. The third part is devoted to some recent developments of spectral methods, such as mixed spectral methods, combined spectral methods and spectral methods on the surface.

Over the second half of the 20th century the subject area loosely referred to as numerical analysis of partial differential equations (PDEs) has undergone unprecedented development. At its practical end, the vigorous growth and steady diversification of the field were stimulated by the demand for accurate and reliable tools for computational moulding in physical sciences and engineering, and by the rapid development of computer hardware and architecture. At the more theoretical end, the analytical insight into the underlying stability and accuracy properties of computational algorithms for PDEs was deepened by building upon recent progress in mathematical analysis and in the theory of PDEs. To embark on a comprehensive review of the field of numerical analysis of partial differential equations within a single volume of this journal would have been an impossible task. Indeed, the 16 contributions included here, by some of the foremost world authorities in the subject, represent only a small sample of the major developments. We hope that these articles will, nevertheless, provide the reader with a stimulating glimpse into this diverse, exciting and important field. The opening paper by Thomée reviews the history of numerical analysis of PDEs, starting with the 1928 paper by Courant, Friedrichs and Lewy on the solution of problems of mathematical physics by means of finite differences. This excellent survey takes the reader through the development of finite differences for elliptic problems from the 1930s, and the intense study of finite differences for general initial value problems during the 1950s and 1960s. The formulation of the concept of stability is explored in the Lax equivalence theorem and the Kreiss matrix lemmas.
is made to the introduction of the finite element method by structural engineers, and a description is given of the subsequent development and mathematical analysis of the finite element method with piecewise polynomial approximating functions. The penultimate section of Thomée's survey deals with 'other classes of approximation methods', and this covers methods such as collocation methods, spectral methods, finite volume methods and boundary integral methods. The final section is devoted to numerical linear algebra for elliptic problems. The next three papers, by Bialecki and Fairweather, Hesthaven and Gottlieb and Dahmen, describe, respectively, spline collocation methods, spectral methods and wavelet methods. The work by Bialecki and Fairweather is a comprehensive overview of orthogonal spline collocation from its first appearance to the latest mathematical developments and applications. The emphasis throughout is on problems in two space dimensions. The paper by Hesthaven and Gottlieb presents a review of Fourier and Chebyshev pseudospectral methods for the solution of hyperbolic PDEs. Particular emphasis is placed on the treatment of boundaries, stability of time discretisations, treatment of non-smooth solutions and multidomain techniques. The paper gives a clear view of the advances that have been made over the last decade in solving hyperbolic problems by means of spectral methods, but it shows that many critical issues remain open. The paper by Dahmen reviews the recent rapid growth in the use of wavelet methods for PDEs. The author focuses on the use of adaptivity, where significant successes have recently been achieved. He describes the potential weaknesses of wavelet methods as well as the perceived strengths, thus giving a balanced view that should encourage the study of wavelet methods.

This text introduces numerical methods and shows how to develop, analyze, and use them. Complete MATLAB programs are now available at www.cambridge.org/Moin, and more than 30 exercises have been added. This thorough and practical book is a first course in numerical analysis for new graduate students in engineering and physical science.

This two volume work presents research workers and graduate students in numerical analysis with a state-of-the-art survey of some of the most active areas of numerical analysis. The work arises from a Summer School covering recent trends in the subject. The chapters are written by the main lecturers at the School each of whom are internationally renowned experts in their respective fields. This extensive coverage of the major areas of research will be invaluable for both theoreticians and practitioners. This volume covers research in the numerical analysis of nonlinear phenomena: evolution equations, free boundary problems, spectral methods, and numerical methods for dynamical systems, nonlinear stability, and differential equations on manifolds.

This book is a pedagogical presentation of the application of spectral and pseudospectral methods to kinetic theory and quantum mechanics. There are additional applications to astrophysics, engineering, biology and many other fields. The main objective of this book is to provide the basic concepts to enable the use of spectral and pseudospectral methods to solve problems in diverse fields of interest and to a wide audience. While spectral methods are generally based on Fourier Series or Chebyshev polynomials, non-classical polynomials and associated quadratures are used for many of the applications presented in the book. Fourier series methods are summarized with a discussion of the resolution of the Gibbs phenomenon. Classical and non-classical quadratures are used for the evaluation of integrals in reaction dynamics including nuclear fusion, radial integrals in density functional theory, in elastic scattering theory and other applications. The subject matter includes the calculation of transport coefficients in gases and other gas dynamical problems based on spectral and pseudospectral solutions of the Boltzmann equation. Radiative transfer in astrophysics and atmospheric science, and applications to space physics are discussed. The relaxation of initial non-equilibrium distributions to equilibrium for several different systems is studied with the Boltzmann and Fokker-Planck equations. The eigenvalue spectra of the linear operators in the Boltzmann, Fokker-Planck and Schrödinger equations are studied with spectral and pseudospectral methods based on non-classical orthogonal polynomials. The numerical methods referred to as the Discrete Ordinate method, Differential Quadrature, the Quadrature Discretization method, the Discrete Variable representation, the Lagrange mesh method, and others are discussed and compared. MATLAB codes are provided for most of the numerical results reported in the book - see Link under 'Additional Information' on the the right-hand column.

This is a book about spectral methods for partial differential equations: when to use them, how to implement them, and what can be learned from their of spectral methods has evolved rigorous theory. The computational side vigorously since the early 1970s, especially in computationally intensive of the more spectacular applications are applications in fluid dynamics. Some of the power of these discussed here, first in general terms as examples of the methods have been methods and later in great detail after the specifics covered. This book pays special attention to those algorithmic details which are essential to successful implementation of spectral methods. The focus is on algorithms for fluid dynamical problems in transition,
turbulence, and aerodynamics. This book does not address specific applications in meteorology, partly because of the lack of experience of the authors in this field and partly because of the coverage provided by Haltiner and Williams (1980). The success of spectral methods in practical computations has led to an increasing interest in their theoretical aspects, especially since the mid-1970s. Although the theory does not yet cover the complete spectrum of applications, the analytical techniques which have been developed in recent years have facilitated the examination of an increasing number of problems of practical interest. In this book we present a unified theory of the mathematical analysis of spectral methods and apply it to many of the algorithms in current use.

This book presents the basic algorithms, the main theoretical results, and some applications of spectral methods. Particular attention is paid to the applications of spectral methods to nonlinear problems arising in fluid dynamics, quantum mechanics, weather prediction, heat conduction and other fields. The book consists of three parts. The first part deals with orthogonal approximations in Sobolev spaces and the stability and convergence of approximations for nonlinear problems, as the mathematical foundation of spectral methods. In the second part, various spectral methods are described, with some applications. It includes Fourier spectral method, Legendre spectral method, Chebyshev spectral method, spectral penalty method, spectral vanishing viscosity method, spectral approximation of isolated solutions, multi-dimensional spectral method, spectral method for high-order equations, spectral-domain decomposition method and spectral multigrid method. The third part is devoted to some recent developments of spectral methods, such as mixed spectral methods, combined spectral methods and spectral methods on the surface.

Lead the reader to a theoretical understanding of the subject without neglecting its practical aspects. The outcome is a textbook that is mathematically honest and rigorous and provides its target audience with a wide range of skills in both ordinary and partial differential equations. -- Book jacket.

This book provides an elementary yet comprehensive introduction to the numerical solution of partial differential equations (PDEs). Used to model important phenomena, such as the heating of apartments and the behavior of electromagnetic waves, these equations have applications in engineering and the life sciences, and most can only be solved approximately using computers. Numerical Analysis of Partial Differential Equations Using Maple and MATLAB provides detailed descriptions of the four major classes of discretization methods for PDEs (finite difference method, finite volume method, spectral method, and finite element method) and runnable MATLAB® code for each of the discretization methods and exercises. It also gives self-contained convergence proofs for each method using the tools and techniques required for the general convergence analysis but adapted to the simplest setting to keep the presentation clear and complete. This book is intended for advanced undergraduate and early graduate students in numerical analysis and scientific computing and researchers in related fields. It is appropriate for a course on numerical methods for partial differential equations.

This book focuses on the constructive and practical aspects of spectral methods. It rigorously examines the most important qualities as well as drawbacks of spectral methods in the context of numerical methods devoted to solve non-standard eigenvalue problems. In addition, the book also considers some nonlinear singularly perturbed boundary value problems along with eigenproblems obtained by their linearization around constant solutions. The book is mathematical, poising problems in their proper function spaces, but its emphasis is on algorithms and practical difficulties. The range of applications is quite large. High order eigenvalue problems are frequently beset with numerical ill conditioning problems. The book describes a wide variety of successful modifications to standard algorithms that greatly mitigate these problems. In addition, the book makes heavy use of the concept of pseudospectrum, which is highly relevant to understanding when disaster is imminent in solving eigenvalue problems. It also envisions two classes of applications, the stability of some elastic structures and the hydrodynamic stability of some parallel shear flows. This book is an ideal reference text for professionals (researchers) in applied mathematics, computational physics and engineering. It will be very useful to numerically sophisticated engineers, physicists and chemists. The book can also be used as a textbook in review courses such as numerical analysis, computational methods in various engineering branches or physics and computational methods in analysis.

This book focuses on the constructive and practical aspects of spectral methods. It rigorously examines the most important qualities as well as drawbacks of spectral methods in the context of numerical methods devoted to solve non-standard eigenvalue problems. In addition, the book also considers some nonlinear singularly perturbed boundary value problems along with eigenproblems obtained by their linearization around constant solutions. The book is mathematical, poising problems in their proper function spaces, but its emphasis is on algorithms and practical difficulties. The range of applications is quite large. High order eigenvalue problems are frequently beset with numerical ill conditioning problems. The book
describes a wide variety of successful modifications to standard algorithms that greatly mitigate these problems. In addition, the book makes heavy use of the concept of pseudospectrum, which is highly relevant to understanding when disaster is imminent in solving eigenvalue problems. It also envisions two classes of applications, the stability of some elastic structures and the hydrodynamic stability of some parallel shear flows. This book is an ideal reference text for professionals (researchers) in applied mathematics, computational physics and engineering. It will be very useful to numerically sophisticated engineers, physicists and chemists. The book can also be used as a textbook in review courses such as numerical analysis, computational methods in various engineering branches or physics and computational methods in analysis.


This well-written book explains the theory of spectral methods and their application to the computation of viscous incompressible fluid flow, in clear and elementary terms. With many examples throughout, the work will be useful to those teaching at the graduate level, as well as to researchers working in the area.

Since the publication of "Spectral Methods in Fluid Dynamics" 1988, spectral methods have become firmly established as a mainstream tool for scientific and engineering computation. The authors of that book have incorporated into this new edition the many improvements in the algorithms and the theory of spectral methods that have been made since then. This latest book retains the tight integration between the theoretical and practical aspects of spectral methods, and the chapters are enhanced with material on the Galerkin with numerical integration version of spectral methods. The discussion of direct and iterative solution methods is also greatly expanded.

This extensively updated edition includes new chapters on emerging subject areas including geometric numerical integration, spectral methods and conjugate gradients.

This paper reviews some operators that are used in the numerical analysis of spectral and spectral element methods. We motivate the introduction of these different operators and sketch their approximation properties. Finally we apply them to derive optimal error estimates for spectral type approximations of the solution of elliptic partial differential equations.

This monograph gives a mathematical analysis of spectral methods for mixed initial-boundary value problems. Spectral methods have become increasingly popular in recent years, especially since the development of fast transform methods, with applications in numerical weather prediction, numerical simulations of turbulent flows, and other problems where high accuracy is desired for complicated solutions. The development of a mathematical theory is given that explains why spectral methods work and how well they work.

This book deals with the application of spectral methods to problems of uncertainty propagation and quantification in model-based computations. It specifically focuses on computational and algorithmic features of these methods which are most useful in dealing with models based on partial differential equations, with special attention to models arising in simulations of fluid flows. Implementations are illustrated through applications to elementary problems, as well as more elaborate examples selected from the authors' interests in incompressible vortex-dominated flows and compressible flows at low Mach numbers. Spectral stochastic methods are probabilistic in nature, and are consequently rooted in the rich mathematical foundation associated with probability and measure spaces. Despite the authors' fascination with this foundation, the discussion only alludes to those theoretical aspects needed to set the stage for subsequent applications. The book is authored by practitioners, and is primarily intended for researchers or graduate students in computational mathematics, physics, or fluid dynamics. The book assumes familiarity with elementary methods for the numerical solution of time-dependent, partial differential equations; prior experience with spectral methods is naturally helpful though not essential. Full appreciation of elaborate examples in computational fluid dynamics (CFD) would require familiarity with CFD and in some cases delicate, features of the associated numerical methods. Besides these shortcomings, our aim is to treat algorithmic and computational aspects of spectral stochastic methods with details sufficient to address and reconstruct all but those highly elaborate examples.

A logically organized advanced textbook, which turns the reader into an active participant by asking questions, hinting, giving direct recommendations, comparing different methods, and discussing "pessimistic" and "optimistic" approaches to numerical analysis. A advanced students and graduate students majoring in computer science, physics and mathematics will find
A Theoretical Introduction to Numerical Analysis presents the general methodology and principles of numerical analysis, illustrating these concepts using numerical methods from real analysis, linear algebra, and differential equations. The book focuses on how to efficiently represent mathematical models for computer-based study. An accessible yet rigorous mathematical introduction, this book provides a pedagogical account of the fundamentals of numerical analysis. The authors thoroughly explain basic concepts, such as discretization, error, efficiency, complexity, numerical stability, consistency, and convergence. The text also addresses more complex topics like intrinsic error limits and the effect of smoothness on the accuracy of approximation in the context of Chebyshev interpolation, Gaussian quadratures, and spectral methods for differential equations. Another advanced subject discussed, the method of difference potentials, employs discrete analogues of Calderon's potentials and boundary projection operators. The authors often delineate various techniques through exercises that require further theoretical study or computer implementation. By lucidly presenting the central mathematical concepts of numerical methods, A Theoretical Introduction to Numerical Analysis provides a foundational link to more specialized computational work in fluid dynamics, acoustics, and electromagnetism.